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KRZYSZTOF WILDE

APPLICATION OF WAVELET ANALYSIS IN DAMAGE DETECTION AND LOCALIZATION

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CONTENTS

LIST OF SYMBOLS AND ABBREVIATIONS

Symbols

a	– depth of a defect
$a_j[k]$	– one-dimensional discrete approximation
$a_j[k,m]$	– two-dimensional discrete approximation
$Af(u,v,s)$	– angle of the wavelet transform vector
$\mathbf{A}(\omega)$	– accelerance
b	– bias of neural network
B	– width
B_1	– distance from a support to a defect along width
B_r	– width of a defect
$\mathbf{B}(\mathrm{s})$	– system matrix
$\mathbf{C}$	– damping matrix
$\overline{\mathbf{C}}$	– modal damping matrix
$d_j[k]$	– one-dimensional discrete wavelet coefficient
$d_j^i\,[k,m]$	– two-dimensional discrete wavelet coefficient
D_1, D_2	– diameters
E	– Young's modulus (chapter 4)
E	– error minimized by training algorithm (chapter 5)
f	– frequency
f_a	– pseudo-frequency of wavelet transform
f_c	– centre frequency of a wavelet
$f(t)$	– one-dimensional time signal
$f(x)$	– one-dimensional space signal
$\mathbf{f}(t)$	– force vector
$f(x,y)$	– two-dimensional space signal
F	– value of concentrated static load
$F(\omega)$	– Fourier transform of one-dimensional signal
g	– high-pass filter
$g_{u,\zeta}(t)$	– window function
G_{FF}	– autospectrum of the force
G_{FX}	– cross spectrum between the response and the force
G_{XF}	– cross spectrum between the force and the response
G_{XX}	– autospectrum of the response
h	– low-pass filter
H	– height
H_1	– distance from a support to a defect along height
H_r	– height of a defect
$H(\omega)$	– frequency response function
$H_1(\omega)$, $H_2(\omega)$	– estimators of frequency response function
$\mathbf{H}(\mathrm{s})$	– transfer function matrix
$\mathbf{H}(\omega)$	– frequency response function matrix (receptance)

i – imaginary unit
K – number of neurons in output layer
$\mathbf{K}$ – stiffness matrix
$\overline{\mathbf{K}}$ – modal stiffness matrix
L – length
L_1 – distance from a support to a defect along length
L_2 – distance from the free end of a beam to the load
L_r – length of a defect
$\mathbf{L}^2(\mathrm{R})$ – Hilbert space of measurable, square-integrable one-dimensional functions
$\mathbf{L}^2(\mathrm{R}^2)$ – Hilbert space of measurable, square-integrable two-dimensional functions
m – number of modes
mse – mean sum of squares of the network errors
$msereg$ – mean squared error with regularization performance function
msw – mean of the sum of squares of the network weights and biases
M – number of neurons in hidden layer
$Mf(u,v,s)$ – modulus wavelet transform of two-dimensional signal
$\mathbf{M}$ – mass matrix
$\overline{\mathbf{M}}$ – modal mass matrix
n – number of vanishing moments
n_d – number degree of freedom
net – net function
o – output of neural network
p – pole
P – number of patterns
$P_{\mathbf{V}_j} f$ – orthogonal projection of function $f(x)$ on space $\boldsymbol{V}_\mathrm{j}$
$P_{\mathbf{W}_j} f$ – orthogonal projection of function $f(x)$ on space $\boldsymbol{W}_\mathrm{j}$
$\mathbf{q}(t)$ – modal coordinate vector
R – number of neurons in input layer
$\mathbf{R}(\mathrm{s})$ – residue matrix
R – real numbers
s – scale parameter
s – Laplace variable (section 3.1.2)
$Sf(u,\zeta)$ – windowed Fourier transform of one-dimensional signal
t – time
t_k – scaling factor
t_r – threshold value
T – sampling period
u – translation parameter (position)
u_0 – beam displacement under dead load
u_F – beam displacement under concentrated static load
$\mathbf{u}$ – eigen vector
$\mathbf{U}$ – modal matrix
$\boldsymbol{V}_j$ – approximations space
w – weight of neural network
$Wf(u,s)$ – continuous wavelet transform of one-dimensional signal
$Wf(u,v,s)$ – continuous wavelet transform of two-dimensional signal

$\boldsymbol{W}_j$	– details space
x	– space coordinate
$x(t)$	– displacement signal
$\mathbf{x}(t)$	– displacement vector
$X(\omega)$	– Fourier transform of displacement signal
y	– space coordinate
z	– input of neural network
Z	– integers
γ	– performance ratio
η	– learning rate
$\theta(x)$	– one-dimensional smoothing function
$\theta(x,y)$	– two-dimensional smoothing function
ν	– Poisson ratio
ξ	– damping ratio
ρ	– mass density
σ_t	– time interval
σ_ω	– frequency interval
$\phi(x)$	– scaling function
$\phi_{j,k}(x)$	– family of discrete scaling function
$\Psi(\omega)$	– Fourier transform of one-dimensional mother wavelet function
$\psi(x)$	– one-dimensional mother wavelet function
$\psi_{j,k}(x)$	– family of discrete wavelets
$\psi_{u,s}(x)$	– family of wavelets
$\psi^1(x,y)$	– two-dimensional horizontal wavelet function
$\psi^2(x,y)$	– two-dimensional vertical wavelet function
$\psi^3(x,y)$	– two-dimensional diagonal wavelet function
ω	– circular frequency
ω_d	– circular damped frequency
ω_n	– circular natural frequency
$\boldsymbol{\Omega}$	– spectral matrix

Abbreviations

ANN	– artificial neural network
CMYK	– cyan, magenta, yellow, black space of colours
CWT	– continuous wavelet transform
DWT	– discrete wavelet transform
FEM	– finite element method
FRF	– frequency response function
FT	– Fourier transform
GPS	– Global Positioning System
NDT	– non-destructive testing
RGB	– red, green, blue space of colours
SHM	– structural health monitoring
WFT	– windowed Fourier transform
WT	– wavelet transform

Chapter 1

INTRODUCTION

> *To, że matematycy znajdują szczęście w przestrzeniach Banacha, mogłoby być rzeczą dość zrozumiałą, ale dlaczego przestrzenie te pojawiają się tak często, ilekroć chcemy rozszyfrować strukturę rzeczywistego świata? Czy świat jest zbudowany według recept na szczęście matematyków?*
>
> *The fact that mathematicians find happiness in Banach spaces could be quite understandable, but why these spaces appear every time we want to decipher the structure of the real world? Is the world constructed in accordance to formulas for mathematicians' happiness?*
>
> Michał Heller
> *Szczęście w przestrzeniach Banacha, 1997*
> *Happiness in Banach spaces, 1997*

1.1. Damage detection in civil engineering structures

All structures raised by humans have a limited lifespan. They wear out and undergo self-destruction in the course of time. Fatigue, corrosion, dynamic phenomena, overloading and environmental conditions can cause their degradation. In recent years, structural damage detection and health monitoring have emerged as the subject of intensive investigation due to their practical importance. For structures like offshore platforms, dams, transmission towers, bridges, aircraft, etc. (Fig. 1.1) early detection of damage is essential since propagation of defects might lead to a catastrophic failure. Accurate detection of damage is also necessary in structural strengthening or reconstruction.

A damage detection system can have four levels of the defect identification accuracy proposed by Rytter in 1993 [99]:

— level 1: the presence of damage,
— level 2: the geometric location of damage,
— level 3: the quantification of the severity of damage,
— level 4: the prediction of the remaining service life of the structure.

The most common method of a non-destructive assessment of the structure integrity is a routine visual inspection, mandatory for important structures. For example, bridges have to be regularly checked by experienced engineers. Damage detection can be facilitated by non-destructive testing (NDT) based on radiography [37, 103], acoustic emission [77, 91], ultrasonic testing [128], magnetic fields methods [54, 120], eddy current methods [35], etc. Although such diagnostic methods can be effectively applied to damage detection in a few known *a priori* areas in a structure, however, they are impractical for a search of potential damage through all engineering object. Additionally, the mentioned NDT methods do not allow an on-line inspection but they are done at periodic maintenance check.

The further development in the NDT methods leads to so-called “structural health monitoring” (SHM). The structural health monitoring is a sub-discipline of the structural engineering which is focused on non-destructive techniques related to continuous, automatic and real time *in situ* monitoring of physical parameters to detect any changes in structures or their abnormal states. There are two major types of monitored parameters, i.e. the load effects (wind, earthquake, temperature, traffic movements, etc.) and the structural responses (displacements, accelerations, velocities, stresses, strains, etc.) [110]. A typical SHM system includes three major components: a sensor system (seismometers, anemometers, accelerometers, velocity and displacement gauges, Global Positioning Systems, thermometers, etc.), a data processing system (including data acquisition, transmission and storage) and a health evaluating system (including diagnostic algorithms and information management) [56].

The SHM techniques are applied to the structures of a special importance like wind turbines [32], offshore structures [75], aircraft [1, 34, 71] or bridges [57, 58, 79, 85]. SHM of bridges can be represented by the example of Commodore Barry Bridge in Philadelphia [3]. The continuous real-time monitor system has been functioning since 1998 on this cantilevered trough-truss bridge. The 145-channels system measures ambient temperatures and wind speed in three directions at several locations once a second. The displacement sensors are installed on the piers and at the various locations of the structure for measuring the movement history. The system also monitors live load images and the corresponding strains and acceleration responses. The integrated streams of data are transmitted from the bridge data server through Internet for the remote control of data acquisition, viewing, processing and archive [3].

Fig. 1.1. Engineering structures (photographed by M. Rucka)

The vibration-based methods and the wave propagation methods play a significant role in SHM strategies of damage detection. The wave propagation is an extension of the NDT wave testing from the local to global approach of sending waves. The passing of waves through material thickness is extended to methods based on the wave propagation along the structure [50, 82, 113]. Guided ultrasonic waves or guided acoustic Lamb waves are attractive due to their ability of inspecting large-structures with a small number of transducers [92, 106, 112]. Detection of Lamb waves can be also achieved by the use of the optical fibre sensors [56, 112].

The vibration-based methods make use of the vibration structure characteristics like the modal frequencies, modal damping and modal shapes, e.g. [89, 90, 127]. Damage in a structure alters values of the dynamic parameters. The presence of damage in structures results in reduction of stiffness and increase of damping. The reduction of stiffness causes a decrease in the natural frequencies of vibration and modification of the mode shapes. Therefore, the relatively simple vibration measurements of a structure and the information extraction of the natural frequencies, damping or mode shapes from the data make damage detection possible. The earlier works on damage detection by the measurements of the natural frequencies have been presented by Cawley and Adams [13]. The procedures of crack detection using the frequency measurements are given, for example, in references [45, 55] and [84]. The applicability of the natural frequency-based method is limited since even significant damage may cause very small changes in the natural frequencies, particularly in large structures, e.g. [25, 48]. Detection of defects and their location might be performed on the displacement mode shapes and their derivatives [2, 83, 123]. This method is proved to be effective in the case of vibration data obtained from numerical simulations. However, with the noisy experimental data, the success of the technique is significantly affected [69]. An improved identification method based on modal information has been presented by Kim and Stabbs [46]. An extensive review of previous research on the vibration-based methods is given in [22, 23, 26, 28] and [101].

Structural health monitoring can be supported by the artificial neural networks (ANN) to process the output data. The neural networks can play the major role in the recognition of sets of data that are related to damage or failure phenomena in the structures that are monitored [59]. There are two main reasons to apply ANN: extracting hidden information from noisy data and the possibility of automatic operation in real time.

1.2. Wavelet transform application in damage detection

The application of wavelet transforms to a wide variety of problems is so plentiful that they have emerged as the most promising techniques in the past decade. Wavelets help to analyse the variations of values at financial markets [108, 109]. The biologists use them for cell membrane recognition. The physicians can estimate electrocardiogram (ECG) parameters [104] and detect myocardial ischemic events using wavelets [88]. The Federal Bureau of Investigation (FBI) considers wavelet application for storage of 30 million sets of criminal fingerprints [107]. The computer scientists exploit them in image processing like edge recognition, image searching, animation control, image compression and even internet traffic description [65, 102]. Engineers use wavelet transforms for time phenomena study in transient processes in earthquake, wind, ocean and mechanical engineering [5, 33, 36] or dynamic silo flow [76]. The wavelet transform is also very useful in modal parameters identification [53], especially damping [105].

Recently, wavelets have been tested for structural health monitoring and damage detection [40, 47, 111]. The ability to monitor the structural parameters and detect damage at the earliest possible stage becomes an important issue throughout the aircraft, mechanical [86, 114] and civil engineering communities [68]. A crack in a structure introduces singularities to the mode shapes or the static deflection lines. These small defects cannot be identified directly from the structure response, but may be observed on the wavelet transforms since the local abnormalities in the signal cause variations of the wavelet coefficients in the neighbourhood of damage.

The literature on wavelet transforms in the one-dimensional case is very extensive. The applicability of various wavelets in cracks detection in beams has been studied by Douka *et al*. [24], Loutridis *et al*. [61], Quek *et al*. [87], Hong *et al*. [39], Wang and Deng [115], Chang and Chen [15] as well as Gentile and Messina [31, 69]. The frame structures treated by the one-dimensional signals have been analysed by Ovanesova and Suarez [80]. The previous studies have shown very good accuracy and effectiveness of the wavelet transform although most of the investigations were performed on the numerical data without the experimental verification. Gentile and Messina [31, 69], Chang and Chen [15] as well as Hong *et al*. [39] pointed out the importance of taking into account the effect of noise. In their study the theoretical mode shapes have been contaminated by a Gaussian noise. The wavelet analysis showed that the presence of noise can mask damage, particularly for fine scale values.

For a practical application of the wavelet damage detection techniques, research on experimental data is the most important. The applicability of the wavelet damage detection techniques depend on the measurement precision and the sampling distances. Pai and Young [81] used a scanning laser vibrometer for non-contact measurements of the beams velocities. The obtained noise level was estimated to have very small standard deviation going below 1% of the maximum reference value. Additionally, the device allowed the measurements of up to 400 equally spaced points. Hong *et al*. [39] and Douka *et al*. [24] showed that the effectiveness of the wavelets for damage localization is highly limited by the sampling distances. They used the dynamic mode shapes extracted from the acceleration measurements. One accelerometer was kept as a reference input, while the second one was moved along the beam. They performed the measurements in 39 points of the beam. For the wavelet analysis, the signal was oversampled to 390 points by a cubic spline interpolation. Rucka and Wilde [98] used the photogrammetric displacement measurement technique that allowed the high precision measurements of the beam static displacements in 81 points. Although current works show that only relatively large cracks can be detected, the search for structural damage by the wavelets is a promising and developing field of research. The mode shape measurements performed by Rucka and Wilde [96] in 48 points showed effectiveness of wavelet-based damage localization in beams.

Two-dimensional damage detection problems were addressed by Wang and Deng [115]. They analyzed a steel plate with an elliptical hole and subjected to uniform tensile loading. The static displacement field has been determined by an analytical formula and has been considered as an input for the wavelet transform. The location of the crack tip has been found by variation of the Haar wavelet coefficients. Douka *et al*. [25] studied vibrations of a rectangular plate with a crack running parallel to one side of the plate. The wavelet transform has been successfully applied to the analytically determined eignefunctions. The cracks of a relative depth from 10% to 50% have been considered. The proposed intensity factor allowed estimation of damage size. The works based on the numerically computed plate mode shapes were presented by Chang and Chen [16] and Rucka and Wilde [93]. The wavelet transforms of the two-dimensional plate problems [16, 25, 93] were addressed by the one-dimensional wavelet analysis since the signal lines at different locations have been treated separately.

Experimental research on plate damage detection has been presented by Rucka and Wilde [95, 118]. The experimental mode shapes of the cantilever plate have been determined by acceleration measurement in one point and impact excitation in 66 points. The relative depth of the introduced rectangular defect was about 19%. The location of damage was determined by Gaussian wavelet with 4 vanishing moments. However, the problem was approached by the one-dimensional wavelet formulation. The two-dimensional formulation of the wavelet transform for plate damage detection is presented by Rucka and Wilde in [97].

1.3. Aim and scope of study

The aim of the research is to verify the applicability of the wavelet analysis in damage identification. The effectiveness of the wavelet transform in damage detection is tested on the numerically and experimentally determined static or dynamic responses of a beam, plate and shell. The improvement of the wavelet-based damage detection method is obtained by the use of the artificial neural network systems.

The work consists of the following chapters:

Chapter 1 reviews the structural health monitoring methods and the wavelet transform application in damage detection.

The wavelet theory is introduced in **Chapter 2**. The definitions of the one-dimensional and two-dimensional wavelet transforms are given. The continuous and discrete wavelet transforms are considered. The examples of the real wavelet functions are given and the ability of wavelet transforms to detect the singularities in the response signals is presented.

Chapter 3 deals with the application of the wavelet theory to damage detection. The experimental procedures to estimate the mode shapes and static deflection lines are described. The wavelet selection is carried out and the best candidates for damage detection are presented. The insight of the differential action of the wavelet transform is discussed.

Chapter 4 examines the applicability of the wavelet transforms to damage detection on the experimental and numerical examples. The beam, plate and shell structures are considered. The location of the defects is searched by the analysis of the spatial variation of the transformed responses

In **Chapter 5** the application of the artificial neural networks to damage prediction based on the wavelet coefficients is shown. The backpropagation algorithm for training a supervised feedforward multilayer neural network is used. The neural network system is considered as the method for the improvement of the efficiency of the damage location technique.

Final conclusions, original elements of the study and the recommendations for the future work are presented in **Chapter 6**.